scattering.[19] Dynamical calculations[11,12] of the π-π scattering amplitude consistently yield a different result: $\alpha_\rho(0)\gtrsim 0.9$. It should be noted, however, that the dynamical calculations of $\alpha_\rho(0)$ are very sensitive to Γ_ρ and were performed for the π-π elastic channel only. The analysis given in this paper is not sensitive to Γ_ρ (see Fig. 4); neither are the p-n charge exchange and π-p scattering analyses.

Let us now examine the consequences of the present analysis for higher photon energies (15–25 BeV). Due to the choice $0.2<\alpha_\rho(0)<0.5$, the ρ-exchange contribution to double pion photoproduction drops slowly with increasing energy. Hence, the differential cross section to photoproduce a high-energy ($K-w_\pi\approx 2$ BeV) charged pion at $\theta_\pi=0°$ will be about 10^{-2} mb/BeV-sr for $K=20$ BeV. This cross section is comparable to the electromagnetic differential cross section (18). Since the energy dependence of photoproduction cross sections is not very well known, it is very speculative to compare these numbers with the Drell cross section. An un-Reggeized π-exchange process contributes a cross section of about 0.3 mb/BeV-sr at $\theta_\pi\approx m_\pi/w_\pi$. Reggeization of this contribution reduces this number by one order of magnitude.[3] Comparison of these cross sections is given in Fig. 10 for $K=20$ BeV, $w_\pi=18$ BeV. As can be seen, detection of the ρ-exchange cross section at $\theta_\pi=0°$ is more complicated for higher energies since the angular resolution needed is one-tenth of a degree, as compared with half a degree at $K=5$ BeV.

The considerations given in this paper may be applied to K photoproduction if the correspondence $\pi\rightarrow K$, $\rho\rightarrow K^*$ is made.

[19] G. Von Dardel, D. Dekkers, R. Mermod, M. Vivargent, G. Weber, and K. Winter, Phys. Rev. Letters 8, 173 (1962).

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K-3π Decay and K_2^0–K_1^0 Mass Difference on Current-Current Picture*

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Some consequences of the current-current picture are considered in connection with K-3π decay and the $K_2^0-K_1^0$ mass difference. It is shown that this picture in boson pole approximation can explain both the decay rates and spectra for the 3π decay modes of K. The $K_2^0-K_1^0$ mass difference is considered in terms of a boson pole approximation and a two-pion intermediate state. The relevant weak-coupling constants are estimated on the current-current picture. It is shown that the sign of the mass difference ($\delta m=\delta m_{K_2^0}-\delta m_{K_1^0}$) is positive, which is in agreement with recent experimental indications. The magnitude is somewhat small but it is not excluded by the present experimental situation.

In this paper we wish to discuss K-3π decay and the $K_2^0-K_1^0$ mass difference on the current-current picture. In an earlier paper,[1] we used this picture in a simple approximation to explain the experimental decay rate of $K_1^0\rightarrow\pi^+\pi^-$ and we also showed that the s-wave amplitudes of hyperon nonleptonic decays can be rather well understood on this picture. We now wish to extend similar considerations to K-3π decay and the $K_2^0-K_1^0$ mass difference. We show that the current-current picture seems to explain these processes as well.

K-3π DECAY

We first consider $K^+\rightarrow\pi^+\pi^+\pi^-$ decay and write its matrix element as

$$M=M_S+M_V,$$

where M_S and M_V denote the s-wave and p-wave parts of the decay amplitude, respectively. We now assume that M_S on the basis of the current-current picture is described by

$$M_S=\frac{G}{\sqrt{2}}[\langle 0|j_\mu{}^A|\pi^+\rangle\langle\pi^+\pi^-|g_\mu{}^A|K^+\rangle+\langle 0|g_\mu{}^A|K^+\rangle\langle\pi^+\pi^-|j_\mu{}^A|\pi^+\rangle],\quad(1)$$

where G is the universal Fermi constant and j^A and g^A are the strangeness-conserving and strangeness-changing axial vector currents, respectively. Denoting the four momenta of K^+, π^+, π^+, and π^- by K, k_1, k_2, and k_3, respectively, we can write (1) as

$$M_S=\frac{G}{\sqrt{2}}[f_\pi q_\mu\langle\pi^+\pi^-|g_\mu{}^A|K^+\rangle+f_K q_\mu'\langle\pi^+\pi^-|j_\mu{}^A|\pi^+\rangle],\quad(2)$$

where $q=K-(k_1+k_3)=k_2$ and $q'=(-k_2)-(k_1+k_3)=-K$ are the relevant momentum transfers and f_π and

* This work was supported by the U. S. Atomic Energy Commission.

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[1] Riazuddin, A. H. Zimerman, and Fayyazuddin, Nuovo Cimento, 32, 1819 (1964).

f_K are the decay constants for π and K, given, respectively, by:

$$f_\pi = (G_A/G)(\sqrt{2}m_N/g_{\pi NN}),\ f_K \approx (m_\pi/m_K)f_\pi. \qquad (3)$$

Assuming now that divergence of g^A is proportional to the kaon field while that of j^A is proportional to the pion field,[2] we get

$$\begin{aligned} q_\mu\langle\pi^+\pi^-|g_\mu{}^A|K^+\rangle &= f_K m_K{}^2\langle\pi^+\pi^-|K|K^+\rangle \\ q_\mu'\langle\pi^+\pi^-|j_\mu{}^A|\pi^+\rangle &= f_\pi m_\pi{}^2\langle\pi^+\pi^-|\pi|\pi^+\rangle. \end{aligned} \qquad (4)$$

This is equivalent to the boson pole approximation described by Feynman diagrams[3] shown in Fig. 1, so that we get from (2) and (4),

$$\begin{aligned} M_S &= \frac{G}{\sqrt{2}}\left[\frac{32\pi\lambda}{m_K{}^2-m_\pi{}^2}f_\pi f_K m_K{}^2 + \frac{32\pi\lambda}{m_\pi{}^2-m_K{}^2}f_\pi f_K m_\pi{}^2\right] \\ &= \frac{G}{\sqrt{2}}32\pi\lambda f_\pi f_K, \end{aligned} \qquad (5)$$

where λ is defined by the unitary symmetric Lagrangian:

$$4\pi\lambda[\boldsymbol{\pi}\cdot\boldsymbol{\pi}+2\bar{K}K+\eta\eta]^2. \qquad (6)$$

We have to still symmetrize (5) over the two like pions and when we do that we obtain

$$M_S = G(32\pi\lambda)f_K f_\pi. \qquad (7)$$

Now, since experimentally the spectrum of unlike pions in $K^+\to 3\pi$ decay deviates only slightly from the statistical distribution (a fact which will also be borne out by our calculation of the p-wave part M_V to be discussed below), the decay rate for $K^+\to\pi^+\pi^+\pi^-$ is determined almost entirely by M_S and is therefore given by

$$\begin{aligned} R &= |M_S{}^2|\frac{Q^2}{3\sqrt{3}2^7\pi^2 m_K} \\ &= \frac{8}{3\sqrt{3}}\lambda^2 G^2 f_K{}^2 f_\pi{}^2\frac{Q^2}{m_K}, \end{aligned} \qquad (8)$$

where $Q = m_K - 3m_\pi$. Using $g_{\pi NN}{}^2/4\pi \approx 15$, $G = 10^{-5}/m_N{}^2$, $m_K = 3.5m_\pi$, $G_A = 1.25G$, we obtain from (3) and (8),

$$R = 5.4\times10^7\lambda^2\ \mathrm{sec}^{-1}. \qquad (9)$$

[2] The constant of proportionality for the pion case is $f_\pi m_\pi{}^2$ [see M. Gell-Mann and M. Levy, Nuovo Cimento 6, 705 (1960)]. Similarly for the kaon case the proportionality constant is $f_K m_K{}^2$.

[3] It was pointed out by Hori *et al.* [S. Hori, S. Oneda, S. Chilia, and A. Wakasa, Phys. Letters 5, 339 (1963)] that these two diagrams cancel out in exact unitary symmetry limit, i.e., when K-π vertex is constant and is the same whether K or π in this transition is on the mass shell. In our method we are essentially using a broken unitary symmetry where the mass dependence of K-π vertex is determined by the current-current picture so that when we take account of which of K or π is on the mass shell, the two diagrams do not cancel. For a similar approach see S. Oneda and Y. S. Kim (unpublished) where, however, mass dependence of this vertex is introduced arbitrarily.

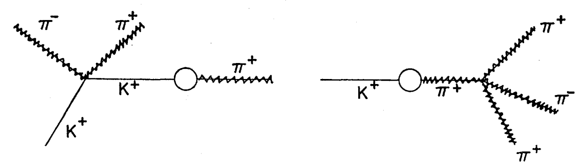

FIG. 1. Feynman diagrams for $K^+\to\pi^+\pi^+\pi^-$ decay in the boson pole approximation.

In order to obtain the experimental value of R, which is about 4.5×10^6 sec^{-1}, we need λ to be about -0.29, which is not inconsistent with the value of λ obtained from the relation $a_0 = -5\lambda$, where a_0 is the s-wave $\pi-\pi$ scattering length and is given by Hamilton *et al.*[4] to be 1.3 ± 0.4. Our value of λ is somewhat larger than the value[4] -0.18 ± 0.5 normally used but is within experimental error.

Let us now discuss the p-wave part of $K^+\to\pi^+\pi^-\pi^+$ decay from the point of view of the current-current picture. The relevant currents here are neutral counterparts of the strangeness-conserving and strangeness-changing vector currents j^V and g^V so that in the same type of approximation as for M_S, we get:

$$\begin{aligned} M_V &= \frac{1}{\sqrt{2}}\langle\pi^+|j_\mu{}^{0V}|\pi^+\rangle\langle\pi^+|g_\mu{}^{0V}|K^+\rangle \\ &= \frac{1}{\sqrt{2}}G(k_3-k_2)_\mu(G^1/G)(K+k_1)_\mu \end{aligned} \qquad (10)$$

+same expression with k_1 and k_2 interchanged, where G^1 is the strength associated with the strangeness-changing vector current and is given by $G^1 = \frac{1}{4}G$. After some manipulation, Eq. (10) can be reduced to the following form[5]:

$$\begin{aligned} M_V &= \frac{G^1}{\sqrt{2}}(-2m_K Qy) \\ &= \frac{G}{4\sqrt{2}}(-2m_K Qy), \end{aligned} \qquad (11)$$

where y is the Dalitz variable $(3T_3-Q)/Q$, T_3 being the kinetic energy of the unlike pion in $K^+\to\pi^+\pi^-\pi^+$ decay. Using (3) and $\lambda\approx-0.29$, we get from (7) and (11):

$$M_V/M_S \approx \tfrac{1}{10}y. \qquad (12)$$

Thus the contribution from the intrinsic structure of primary vector current-current interaction to the spectrum of unlike pion in τ^+ decay is substantial.

It should be noted that $j_\mu{}^A g_\mu{}^A$ can also contribute to

[4] J. Hamilton, P. Menotti, G. C. Oades, and L. J. Vick, Phys. Rev. 128, 1881 (1962).

[5] Similar approach for p-wave part of the decay amplitude has also been proposed by S. Oneda and Y. S. Kim (unpublished).

the p-wave part M_V through the following transitions[6]:

$$K \to \pi \to \rho + \pi \;\searrow\; \pi\pi\,, \qquad K \to K + \rho \;\searrow\; \pi \;\searrow\; 2\pi \tag{A}$$

and[6,7]

$$K \to K^* + \pi \;\searrow\; K + \pi \;\searrow\; \pi\,, \tag{B}$$

where the K-π coupling is due to $j_\mu{}^A g_\mu{}^A$ as follows:

$$f_{K_2{}^0\pi^0} = f_{K^+\pi^+} = f_{K\pi} = (G/\sqrt{2})\langle 0|j_\mu{}^A|\pi\rangle\langle 0|g_\mu{}^A|K\rangle = (G/\sqrt{2})f_\pi f_K m_K{}^2\,, \tag{13}$$

if the kaon is on the mass shell or $(G/\sqrt{2})f_\pi f_K m_\pi{}^2$ if the pion is on the mass shell. This gives $f_{K\pi}{}^2/4\pi \approx 0.12 \times 10^{-13} m_\pi{}^4$ in the former case and $\approx 10^{-14} m_\pi{}^4$ in the latter case. With these values of the K-π coupling constants the contribution from the transition (B) to M_V is negligible, while from the transition (A) it is given by

$$M_V/M_S \approx -\frac{1}{10}(\gamma_{\rho\pi\pi}{}^2/4\pi)\frac{1}{8\lambda}y\,, \tag{14}$$

which is about $\frac{1}{10}$, using[8] $(\gamma_{\rho\pi\pi}{}^2/4\pi) \approx 2$ and $\lambda \approx -0.29$. The contribution to M_V from the s-wave final state π-π interaction as calculated by Khuri and Treiman[9] is given by

$$M_V/M_S \approx \frac{1}{7}(a_2 - a_0)y\,, \tag{15}$$

where we take from Hamilton *et al.*[9] $a_0 \approx 1.7$ and $a_2 = (\frac{2}{5})^{1/2} a_0$. Then (14) and (15) cancel each other. Hence we are left with the contribution (12) and get

$$M \approx M_S(1 + \tfrac{1}{10}y)\,, \tag{16}$$

which gives a good fit to the spectrum of unlike pion in τ^+ decay.[10] However, if a low-mass s-wave resonance exists in the π-π system, it can affect the spectrum (16); but the existence of such a resonance is not yet confirmed experimentally.[11] Note that $j_\mu{}^V g_\mu{}^V$ can also contribute to the K-π vertex used above; however, this involves a continuum of intermediate states which is outside the scope of our model. Our model is based on a simple boson pole approximation in the current-current picture.

We shall not consider the τ' decay mode of K^+ and $K_2{}^0$-3π decays because they are simply related to τ decay by the $\Delta I = \frac{1}{2}$ rule, which is implied in our model and which fits the data.

To summarize, we have shown that the current-current interaction, calculated on the basis of simple boson pole-approximation model is capable of explaining both the decay rates and spectra for 3π decay modes of K^+ and $K_2{}^0$.

$K_2{}^0$–$K_1{}^0$ MASS DIFFERENCE

We first discuss the contribution of π^0-pole to the $(K_2{}^0 - K_1{}^0)$ mass difference. The π^0-pole contributes to the $K_2{}^0$ self-energy only and its contribution is given by

$$\delta m_{K_2{}^0} = \frac{1}{2m_K}\frac{f_{K\pi}{}^2}{m_K{}^2 - m_\pi{}^2}\,, \tag{17}$$

where $f_{K\pi}$ is the coupling constant for the $K_2{}^0 - \pi^0$ transition. This contribution has been considered by several people.[12] Our aim, however, is to obtain an estimate of $\delta m_{K_2{}^0}$ on the current-current picture by using the value of $f_{K\pi}$ estimated by us on this picture in Eq. (13). We obtain

$$\delta m_{K_2{}^0} \approx 3 \times 10^{-7}\,\mathrm{eV}\,,$$

which is too low by an order of magnitude. We see that in this approximate treatment based on the current-current picture, the contribution of the π^0 pole to the mass difference is very small. This is independent of any possible cancellation with the contribution which the η pole[12] may give in the unitary symmetry model.

Next we consider the contribution of the ρ^0 pole to the mass difference. Again, as discussed by Biswas and Bose,[13] ρ^0 contributes to the $K_2{}^0$ self-energy only because of CP invariance. Its contribution to the $K_2{}^0$ self-energy is given by[13]

$$\begin{aligned}\delta m_{K_2{}^0} &= -\frac{1}{2m_K} f_{K\rho}{}^2 q_\mu [\delta_{\mu\nu} + q_\mu q_\nu / m_\rho{}^2](q^2 + m_\rho{}^2)^{-1} q_\nu \\ &= f_{K\rho}{}^2 \frac{m_K}{2m_\rho{}^2}\,,\end{aligned} \tag{18}$$

where $f_{K\rho}$ is the coupling constant for the $K_2{}^0 - \rho^0$ transition. We can estimate $f_{K\rho}(= f_{K^+\rho^+})$ on the current-current picture and it is given by

$$\begin{aligned} f_{K^+\rho^+} q_\mu &= \frac{1}{\sqrt{2}}\langle 0|j_\mu{}^V|\rho^+\rangle\langle 0|g_\mu{}^A|K^+\rangle \\ &= \frac{1}{\sqrt{2}} f_K f_\rho q_\mu\,,\end{aligned}$$

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[6] M. A. Baqi Bég and P. C. Decelles, Phys. Rev. Letters **8**, 46 (1962).
[7] Riazuddin and Fayyazuddin, Phys. Rev. Letters **7**, 464 (1961).
[8] J. J. Sakurai, *Proceedings of the International School of Physics, "Enrico Fermi," Varenna* (Academic Press, Inc., New York, 1962).
[9] N. N. Khuri and S. B. Treiman, Phys. Rev. **119**, 1115 (1960).
[10] M. Gell-Mann and A. H. Rosenfeld, Ann. Rev. Nucl. Sci. **7**, 407 (1957); M. Ferro-Luzzi, D. H. Miller, J. J. Murray, A. H. Rosenfeld, and R. D. Tripp; Nuovo Cimento **22**, 1087 (1961).
[11] See however, L. M. Brown and P. Singer, Phys. Rev. **133**, B812 (1964).
[12] S. K. Bose, Phys. Letters **2**, 92 (1962).
[13] S. N. Biswas and S. K. Bose, Phys. Rev. Letters **12**, 1777 (1964). Due to a wrong sign of the vector boson propagator, these authors got a negative value for $\delta m_{K_2{}^0}$.

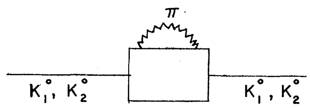

FIG. 2. Feynman diagram for the self-energy of K_1^0 or K_2^0.

where f_ρ is related to G by a Goldberger-Treiman type of relation:

$$f_\rho = (\sqrt{2}Gm_\rho^2)/\gamma_{\rho\pi\pi},$$

$\gamma_{\rho\pi\pi}$ being the same as defined by Sakurai.[8] Hence

$$f_{K\rho} = f_K(Gm^2{}_\rho/\gamma_{\rho\pi\pi}). \tag{19}$$

Using (3) and $G=10^{-5}m_N^{-2}$, $\gamma_{\rho\pi\pi}^2/4\pi \approx 2$, we obtain

$$f_{K\rho}^2 \approx 10^{-13}m_\pi^2,$$

and hence, from (18),

$$\delta m_{K_2^0} = 10^{-6}\text{eV}, \tag{20}$$

so that the mass difference

$$\delta m = \delta m_{K_2^0} - \delta m_{K_1^0} = 10^{-6}\text{eV}.$$

We thus obtain a positive sign for δm which is in accordance with recent experimental indications.[14] However, the magnitude is rather small. We, therefore, discuss other contributions to the mass difference δm, which go beyond a pole approximation. In particular, we consider Feynman diagram for the self-energy of K_1^0 or K_2^0 shown in Fig. 2. In the spirit of the dispersion-theoretic approach, the lowest mass state which replaces the black box shown in Fig. 2 is the pion giving us the Feynman diagram for the self-energy of K_1^0 shown in Fig. 3. The contribution of this diagram to the self-energy of K_1^0 can be easily written down[15] and is given by

$$\delta m_{K_1^0} = \frac{1}{2m_K}\frac{i}{(2\pi)^4}\int d^4q \frac{1}{q^2+m_\pi^2}\frac{1}{(q-p)^2+(m_\pi^2)}F^2(q^2), \tag{21}$$

where $F(q^2)$ is the form factor for $K_1^0 \to \pi^+\pi^-$ decay. This decay has been discussed by us in Ref. 1 and it was shown there that the current-current picture gives the following matrix element for this decay:

$$\begin{aligned}\langle 0|j_\mu{}^A|\pi^+\rangle\langle\pi^-|g_\mu{}^V|K^0\rangle &= f_\pi q_\mu\langle\pi^-|g_\mu{}^V|K^0\rangle \\ &= f_\pi\langle\pi^-|\partial g_\mu{}^V/\partial x_\mu|K^0\rangle.\end{aligned}$$

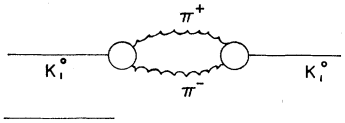

FIG. 3. Feynman diagram for the self-energy of K_1^0 with two pions in the intermediate state.

Since this matrix element is determined by the divergence of $g_\mu{}^V$, which was assumed to be proportional to scalar[16] κ meson ($I=\frac{1}{2}$, $J=0$, $m_\kappa=725$ MeV) in Ref. 1, our current-current picture for $K_1^0 \to \pi^+\pi^-$ is equivalent to the κ pole approximation shown in Fig. 4. Hence, in view of good agreement with experiment for the decay rate of $K_1^0 \to \pi^+\pi^-$ obtained on this picture in Ref. 1, we take the view that the form factor for $K_1^0 \to \pi^-\pi^+$ is dominated by the κ pole and is therefore given by

$$F(q^2) = f_{K_1{}^0\pi^+\pi^-}(m_K^2 - m_\pi^2)/(q^2+m_\kappa^2) \tag{22}$$

where $f_{K_1{}^0\pi^+\pi^-}$ is the decay constant for $K_1^0 \to \pi^+\pi^-$ and is given by[1] $f_{K_1{}^0\pi^+\pi^-}{}^2/4\pi \approx 6\times 10^{-13}m_\pi^2$. Putting

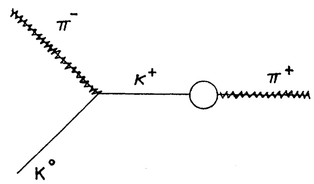

FIG. 4. Feynman diagram for $K_1^0 \to \pi^+\pi^-$ decay in κ pole approximation.

the form factor (22) in (21), we obtain

$$\delta m_{K_1^0} \approx -1.2\times 10^{-6}\text{eV}.$$

If we add the $K_1^0 \to \pi^0\pi^0$ contribution also, where $f_{K_1{}^0\pi^0\pi^0} = (1/2^{1/2})f_{K_1{}^0\pi^+\pi^-}$ on the $\Delta I=\frac{1}{2}$ rule, we get

$$\delta m_{K_1^0} \approx -1.8\times 10^{-6}\text{eV}. \tag{23}$$

Combining (20) and (23), we obtain finally for the mass difference

$$\delta m = \delta m_{K_2^0} - \delta m_{K_1^0} = 2.8\times 10^{-6}\text{eV}. \tag{24}$$

The sign of the mass difference obtained by us in (24) is, as before, in agreement with recent experimental indications.[14] As far as the magnitude is concerned, different experiments have given different values for δm and in fact, at the present time, it can lie in the range[14]:

$$3\times 10^{-6}\text{eV} \leqslant |\delta m| \leqslant 10^{-5}\text{eV}.$$

Unless the experimental situation regarding the magnitude of δm is cleared up, it is premature to say whether the current-current picture as used by us is sufficient to explain the mass difference although the sign seems to be correct.

ACKNOWLEDGMENTS

We are indebted to Professor R. E. Marshak and Dr. S. Okubo for encouragement and advice. One of us (R) would like to thank the U. S. Educational Foundation in Pakistan for the award of a Fulbright travel grant.

[14] International Conference on Fundamental Aspects of Weak Interactions, Brookhaven National Laboratory, Upton, New York, September 1963 (unpublished).

[15] See for instance Riazuddin, Phys. Rev. **114**, 1184 (1959), where a similar problem is discussed for the electromagnetic self-energy of π^+ in dispersion theoretic approach. See also V. Barger and E. Kazes, Phys. Rev. **124**, 279 (1961); and Nuovo Cimento **28**, 394 (1963).

[16] Although the quantum numbers 0^+ for κ are not confirmed, we should expect such a particle on theoretical ground as strangeness-changing vector current is not conserved and its divergence may be connected with the scalar κ.